If $\vec{A}, \vec{B}$ are differentiable vector function of a scalar t then

1. $\frac{d}{dt}(\vec{A}+\vec{B}) = \frac{d\vec{A}}{dt} + \frac{d\vec{B}}{dt}$
2. $\frac{d}{dt}(\vec{A}\cdot\vec{B}) = \vec{A}\cdot\frac{d\vec{B}}{dt} + \frac{d\vec{A}}{dt}\cdot\vec{B}$
3. $\cancel{\frac{d}{dt}(\vec{A}\times\vec{B}) = \vec{A}\times\frac{d\vec{B}}{dt} + \frac{d\vec{A}}{dt}\times\vec{B}}$

1. $\frac{d}{dt}(\vec{A}+\vec{B}) = \frac{d\vec{A}}{dt} + \frac{d\vec{B}}{dt}$

Let $\vec{A} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$
$\vec{B} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$
$\left\{ a_1, a_2, a_3, b_1, b_2, b_3 \text{ functions of } t \right\}$

Then $\vec{A}+\vec{B} = (a_1+b_1)\hat{i} + (a_2+b_2)\hat{j} + (a_3+b_3)\hat{k}$

So $\frac{d}{dt}(A+B) = \left(\frac{da_1}{dt} + \frac{db_1}{dt}\right)\hat{i} + \left(\frac{da_2}{dt} + \frac{db_2}{dt}\right)\hat{j} + \left(\frac{da_3}{dt} + \frac{db_3}{dt}\right)\hat{k}$

$= \frac{da_1}{dt}\hat{i} + \frac{da_2}{dt}\hat{j} + \frac{da_3}{dt}\hat{k} + \frac{db_1}{dt}\hat{i} + \frac{db_2}{dt}\hat{j} + \frac{db_3}{dt}\hat{k}$

$= \frac{d}{dt}(a_1\hat{i} + a_2\hat{j} + a_3\hat{k}) + \frac{d}{dt}(b_1\hat{i} + b_2\hat{j} + b_3\hat{k})$

$= \frac{d\vec{A}}{dt} + \frac{d\vec{B}}{dt}$

2. Let $\vec{A} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$

$\vec{B} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$

~~$\vec{A}\cdot\vec{B} =$~~ Where $a_1, b_1, a_2, b_2, a_3, b_3$ are functions of t.

$$\vec{A}\cdot\vec{B} = a_1b_1 + a_2b_2 + a_3b_3$$

$$\frac{d}{dt}\vec{A}\cdot\vec{B} = \frac{d}{dt}(a_1b_1) + \frac{d}{dt}(a_2b_2) + \frac{d}{dt}(a_3b_3)$$

$$= \frac{da_1}{dt}b_1 + a_1\frac{db_1}{dt} + \frac{da_2}{dt}b_2 + a_2\frac{db_2}{dt} + \frac{da_3}{dt}b_3 + a_3\frac{db_3}{dt}$$

$$= \frac{da_1}{dt}b_1 + \frac{da_2}{dt}b_2 + \frac{da_3}{dt}b_3 + a_1\frac{db_1}{dt} + a_2\frac{db_2}{dt} + a_3\frac{db_3}{dt}$$

$$= \frac{d\vec{A}}{dt}\cdot\vec{B} + \vec{A}\cdot\frac{d\vec{B}}{dt}$$

Alternatively, let at $t+\Delta t$, $\vec{A}$ changes to $\vec{A}+\Delta\vec{A}$ and $\vec{B}$ changes to $\vec{B}+\Delta\vec{B}$

~~So $\frac{d}{dt}(\vec{A}\cdot\vec{B}) = \frac{(A+\Delta A)\cdot(B+\Delta B) - AB}{}$~~

$$\text{So } \frac{d}{dt}(\vec{A}\cdot\vec{B}) = \lim_{\Delta t\to 0}\frac{(\vec{A}+\Delta\vec{A})\cdot(\vec{B}+\Delta\vec{B}) - \vec{A}\cdot\vec{B}}{\Delta t}$$

$$= \lim_{\Delta t\to 0}\frac{\Delta\vec{A}\cdot\vec{B} + \vec{A}\cdot\Delta\vec{B} + \Delta\vec{A}\cdot\Delta\vec{B}}{\Delta t}$$

$$= \lim_{\Delta t\to 0}\frac{\Delta\vec{A}}{\Delta t}\cdot\vec{B} + \lim_{\Delta t\to 0}\vec{A}\cdot\frac{\Delta\vec{B}}{\Delta t} + \lim_{\Delta t\to 0}\frac{\Delta A\cdot\Delta B}{\Delta t\cdot\Delta t}\Delta t$$

$$= \frac{d\vec{A}}{dt}\cdot\vec{B} + \vec{A}\cdot\frac{d\vec{B}}{dt} + \lim_{\Delta t\to 0}\frac{\Delta A}{\Delta t}\cdot\lim_{\Delta t\to 0}\frac{\Delta B}{\Delta t}\lim_{\Delta t\to 0}\Delta t$$

$$= \frac{d\vec{A}}{dt}\cdot\vec{B} + \vec{A}\cdot\frac{d\vec{B}}{dt} + \frac{dA}{dt}\cdot\frac{dB}{dt}\lim_{\Delta t\to 0}\Delta t$$

$$= \frac{d\vec{A}}{dt}\cdot\vec{B} + \vec{A}\cdot\frac{d\vec{B}}{dt} + \frac{d\vec{A}}{dt}\cdot\frac{d\vec{B}}{dt}0 = \frac{d\vec{A}}{dt}\cdot\vec{B} + \vec{A}\cdot\frac{d\vec{B}}{dt}$$